Tap The Sun

The SolarTrak®
Controller System

Pro-Active Sun Tracking and Peripheral System Control

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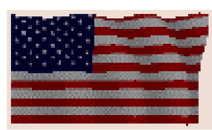


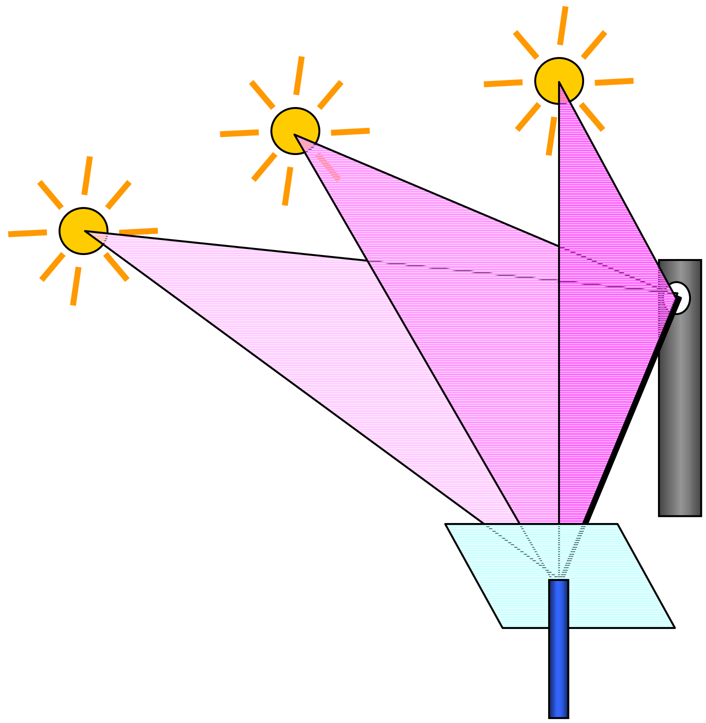
Designed and Built in the U.S.A.

Development consultation provided by:
A.B. Maish, Sandia National Laboratories, Albuquerque, NM.

Abstract

The Sun can be reflected to a single, unmoving target position by the action of a computer-based motion controller using mathematical equations to relate the mechanical position of a mirror to the combination of the Sun's position and a fixed target receiver. The geometrical and mathematical relationships involved are diagramed and defined.

Plane of Reflection



As the Earth rotates, producing the apparent motion of the Sun through the sky, the three objects: the Sun, the Receiver (target) and the Heliostat (actually their center points) define the plane of reflection. The apex of this rotating triangle is the Sun, while the base of the triangle is fixed between the target and heliostat. It is in this plane that the resultant heliostat position vector must lay halfway between the Sun and the target to properly reflect the sunbeam to a fixed target.

Though apparently somewhat extreme, the spherical coordinate system is the simplest to use for the necessary computations. The algebra involved in manipulating the resulting equations is much less complicated than when rectangular coordinates are used.

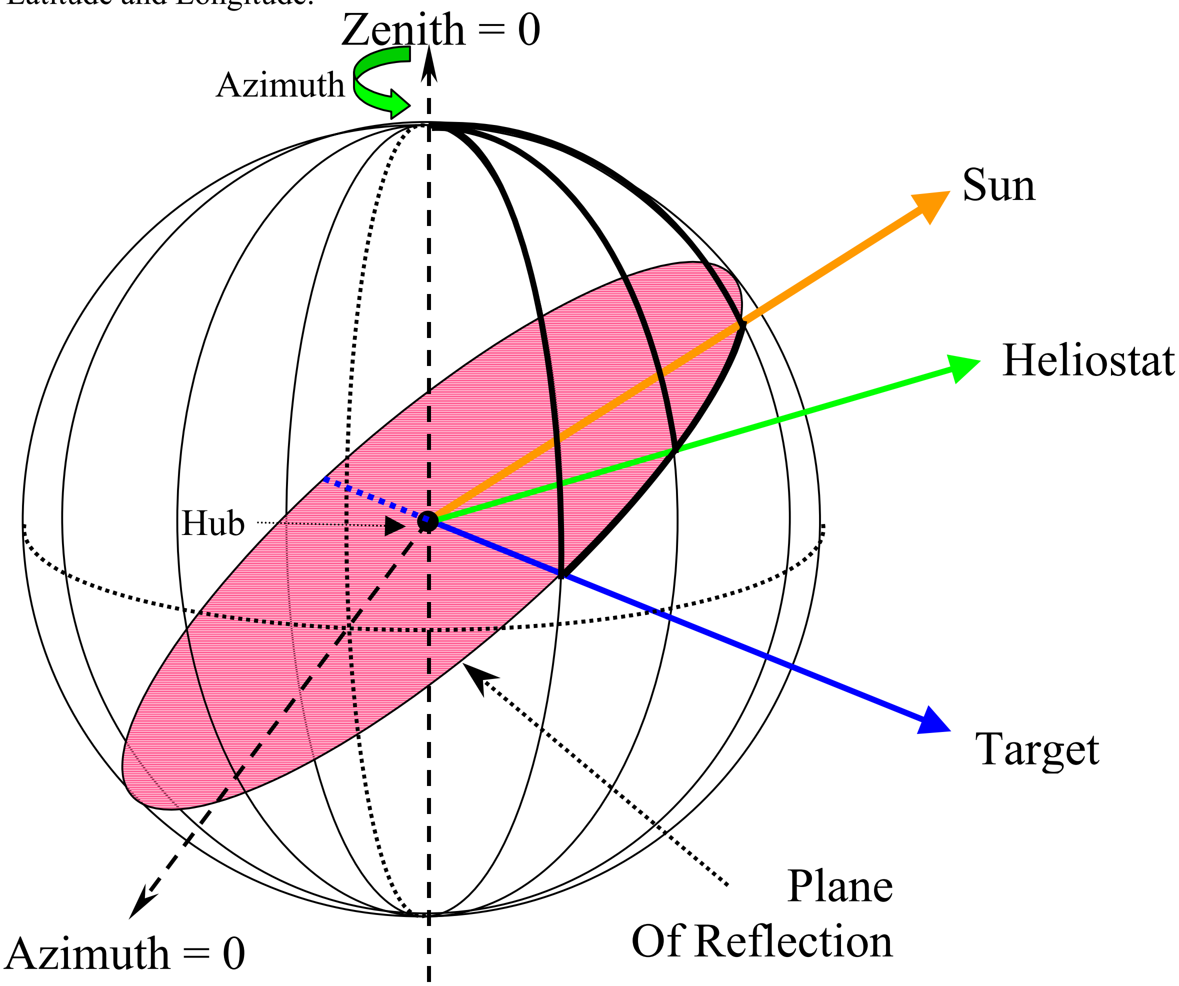
When setting up the diagrams for working through the problem, the heliostat is used as the center of the sphere. From that vantage point the position vector of the Sun can be computed from equations* using the local solar time, date, time zone, latitude and longitude of the heliostat pedestal. The target position vector is constant since neither the heliostat pedestal nor the target moves about although there is some target vector offset due to the motion of the center of the mirror when the surface of the mirror does not lay at the center of rotation.

Due to the extreme distance of the Sun, its apparent angular position does not change when there is translational motion of a few feet or even a mile, however, small translations of the same few feet will produce noticeable changes in the apparent target vector at the finite distances found in a solar field. A translation of 3 feet will produce a target offset of just over 0.2 degrees at 800 feet, which is more than enough to be observable and affect the beam position on the target.

* See Appendix A.

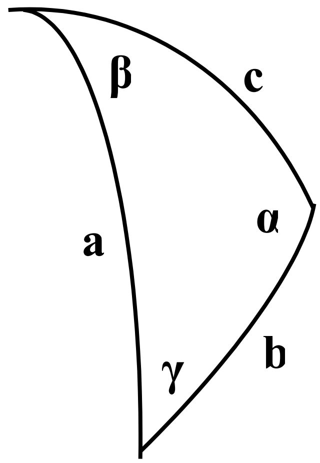
The following diagram highlights the 'working triangle', actually a segment of the surface of a sphere that will be used to build the equations of reflection. Zenith is a rotation about the center of the sphere and constitutes a 'side' measurement of the triangle though actually an angular value. Corner angles, used in the intermediate calculations, are rotations about a particular vector emanating from the center of the sphere and piercing the surface. Azimuth is a special case of rotation about a vector piercing the surface of the sphere where Zenith equals zero. Zenith can be converted to a 'normal' reference by the conversion: Elevation (in degrees) equals 90 minus Zenith.

To relate this 'free-body' diagram to reality, the vector annotated as 'Zenith = 0' is a vector emanating from the center of the Earth and piercing the surface at a particular Latitude and Longitude.



In manipulating the values indicated by the 'working triangle' it is necessary to enlist the use of the law of Sines and the law of Cosines. These geometric laws relate the 'sides' of a triangle and the opposing subtended angles. By systematic usage of these laws, the unknown quantities initially defined can be algebraically calculated such that the proper heliostat position vector can be evaluated for any given Sun position vector given a particular target vector. The plane of reflection rotates about the target vector, its orientation defined by the Sun position vector.

The law of Sines states that each of the three ratios of a side and its opposing subtended angle are equal. The law of Cosines defines the functional relationship between a 'side' or angle and the other two 'sides' or angles and its opposing counterpart.



Law of Sines

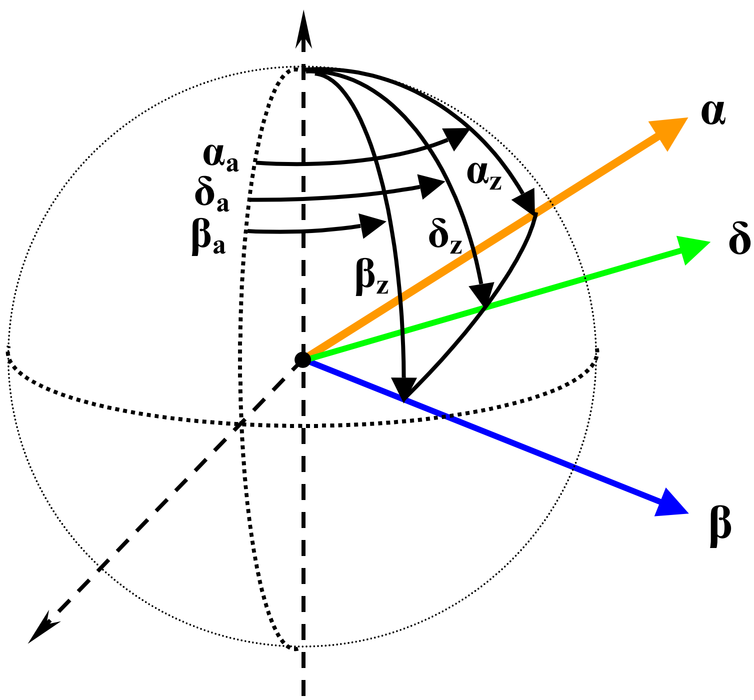
$$\frac{\sin \alpha}{\sin a} = \frac{\sin \beta}{\sin b} = \frac{\sin \gamma}{\sin c}$$

Law of Cosines

$$\cos \alpha = -\cos \beta \cos \gamma + \sin \beta \sin \gamma \cos a$$

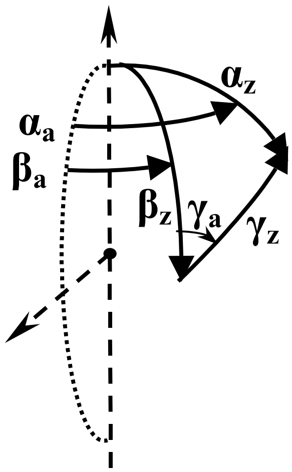
$$\cos a = \cos b \cos c + \sin b \sin c \cos \alpha$$

In each case, one of these two relationships can be used to determine an unknown quantity involved in the reflection equation. There are six starting quantities. In the fundamental reflection equations, the Sun position vector (**α**) and the target vector (**β**) are known and the heliostat position (**δ**) vector must be determined.



In the case of establishing the target vector (targeting), the Sun position vector and heliostat vector are known and the target vector must be determined.

In the first step, the angle between the Sun and the target, γ_z, must be determined so that the halfway point can ultimately be computed. This angle is designated with a 'Z' subscript because it is a rotation about the center of the sphere. The intermediate angle, γ_a, is so designated because it is a rotation about a radial vector at the surface.



Using law of Cosines:

$$\mathbf{Cos}\ \gamma_z = \mathbf{Cos}\ \alpha_z\ \mathbf{Cos}\ \beta_z + \mathbf{Sin}\ \alpha_z\ \mathbf{Sin}\ \beta_z\ \mathbf{Cos}\ (\alpha_a - \beta_a)$$

Then evaluate $\mathbf{Cos^{-1}}\ (\mathbf{Cos}\ \gamma_z)$ to get γ_z.

Using the law of Sines:

$$\frac{\mathbf{Sin}\ (\alpha_a - \beta_a)}{\mathbf{Sin}\ \gamma_z} = \frac{\mathbf{Sin}\ \gamma_a}{\mathbf{Sin}\ \alpha_z}$$

Solve for: $\mathbf{Sin}\ \gamma_a = \mathbf{Sin}\ \alpha_z\ \dfrac{\mathbf{Sin}\ (\alpha_a - \beta_a)}{\mathbf{Sin}\ \gamma_z}$

And evaluate $\mathbf{Sin^{-1}}\ (\mathbf{Sin}\ \gamma_a)$ to get γ_a.

The second step is to derive the heliostat vector given the common angle γ_a and the intermediate value γ_z, which is divided by **2** to get the half-angle of reflection.

Using law of Cosines:

$$\mathbf{Cos}\ \delta_z = \mathbf{Cos}\ \beta_z\ \mathbf{Cos}\left(\frac{\gamma_z}{2}\right) + \mathbf{Sin}\ \beta_z\ \mathbf{Sin}\left(\frac{\gamma_z}{2}\right)\mathbf{Cos}\ (\gamma_a)$$

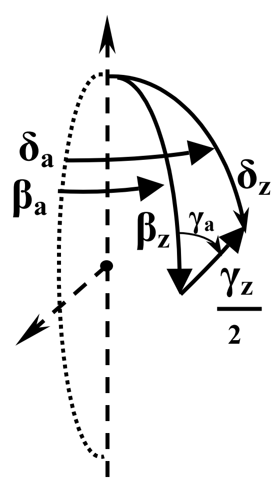
Then evaluate $\mathbf{Cos^{-1}}\ (\mathbf{Cos}\ \delta_z)$ to get δ_z.

Using the law of Sines:

$$\frac{\mathbf{Sin}\ (\delta_a - \beta_a)}{\mathbf{Sin}\left(\frac{\gamma_z}{2}\right)} = \frac{\mathbf{Sin}\ \gamma_a}{\mathbf{Sin}\ \delta_z}$$

Solve for: $\mathbf{Sin}\ (\delta_a - \beta_a) = \mathbf{Sin}\left(\dfrac{\gamma_z}{2}\right)\dfrac{\mathbf{Sin}\ \gamma_a}{\mathbf{Sin}\ \delta_z}$

And evaluate $\delta_a = \mathbf{Sin^{-1}}\left[\mathbf{Sin}\left(\dfrac{\gamma_z}{2}\right)\dfrac{\mathbf{Sin}\ \gamma_a}{\mathbf{Sin}\ \delta_z}\right] + \beta_a$



Heliostat Mirror Offset Corrections

The motion of the mirror about its azimuth and elevation pivots moves the focal line through a sinusoidal pattern across the target due to the offset of the mirror surface from the rotational centerlines usually required by inexpensive structural systems. This pattern is predictable though not entirely determinant (second order angular effects apply) and can be corrected with a minimum of source code overhead and data collection.

Using the following diagrams to illustrate a typical geometry and identify parameters, a mathematical model can be derived that will work easily with the SolarTrak® control code.

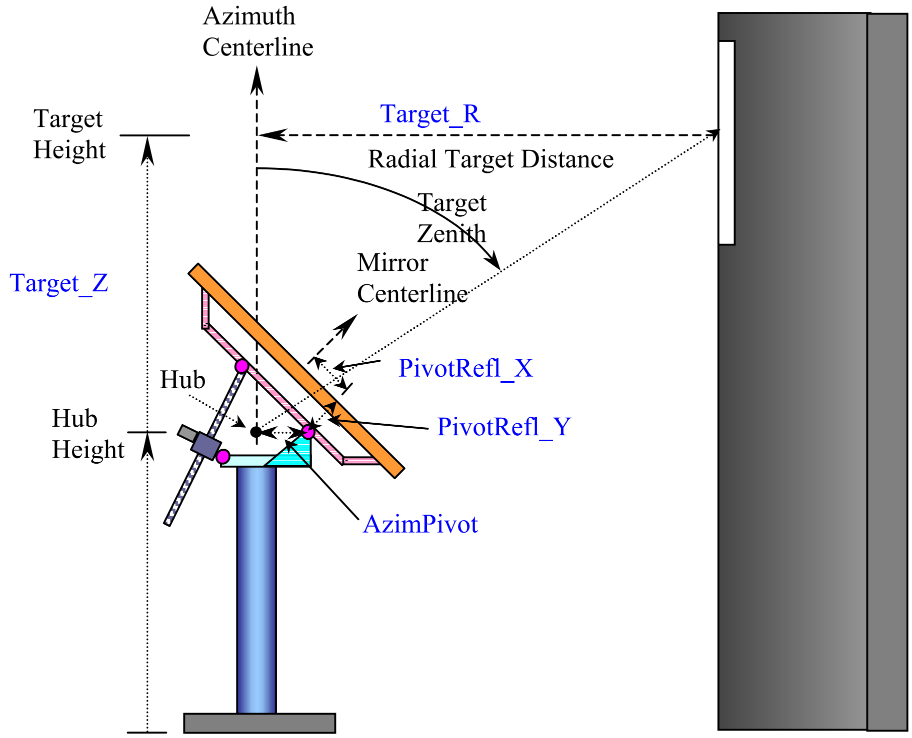


Figure 1: Elevation View – Mirror Position

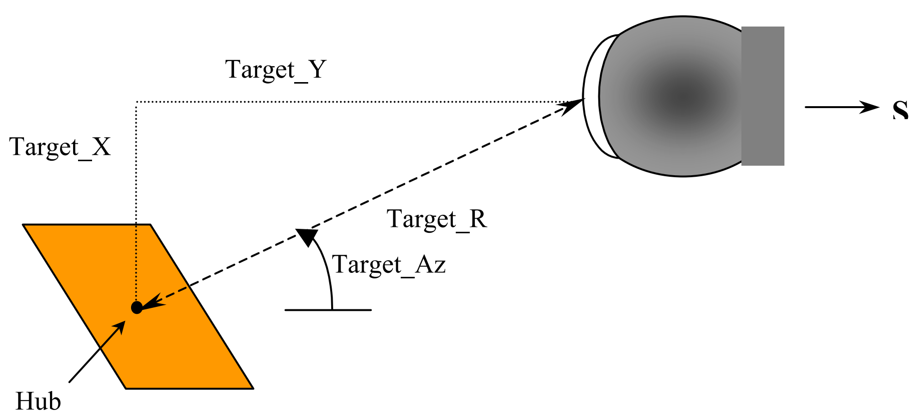


Figure 2: Top View – Mirror Offset

Although the traditional notation for heliostat (pedestal) and tower position has been in X-Y-Z coordinates, the SolarTrak requires only the relative bearing of the target to the HUB (defined as the intersection of the Azimuth center-of-rotation and the vertical height of the Elevation axis-of-rotation).

The location of the target is defined as the Azimuth bearing of the center of the target plus the Zenith depression from vertical. This vector offset is stored in the semi-permanent (reprogrammable) area of the SolarTrak® memory. The normal interpretation of this offset would be from the center of focus of the mirror that would ideally coincide with the HUB.

As reality dictates, the actual focal line is subject to the sinusoidal translation effects induced by the offsets: AzimPivot_R, PivotRefl_R, PivotRefl_Z, Target_R and Target_Z, while the system rotates through its range of motion.

It is not necessary to keep all the raw parameters in the SolarTrak® memory, only the resultant vectors. Target_R is the result of Target_X and Target_Y and can be further combined with Target_Z to yield the actual distance of the target from the hub. The mirror center can also be represented as a vector with a magnitude and offset angle.

Once these values are produced and entered into the SolarTrak® memory, the translation error can be computed as a function of the current mechanical position at any given moment and a correction angle computed for the Azimuth and Zenith target bearing. Since the primary corrections are relatively small, secondary corrections should not be necessary but could be accomplished with iterative calls to the correction subroutine.

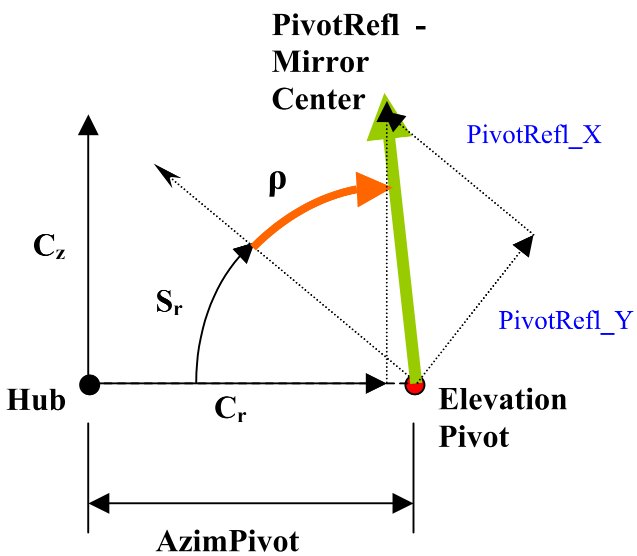


Figure 3: Side Projection of Rotating Elevation Pivot

Cr and **Cz** are the resultant components of the mirror center-of-focus based on the current reflective half-angle zenith component, **Sr**, the **PivotRefl** vector and the fixed angular offset of the center from the elevation pivot, **ρ**.

The zenith component of the offset, dT_Z, is based on the projection of the mirror-offset vector, **C**, with respect to the ideal beam vector.

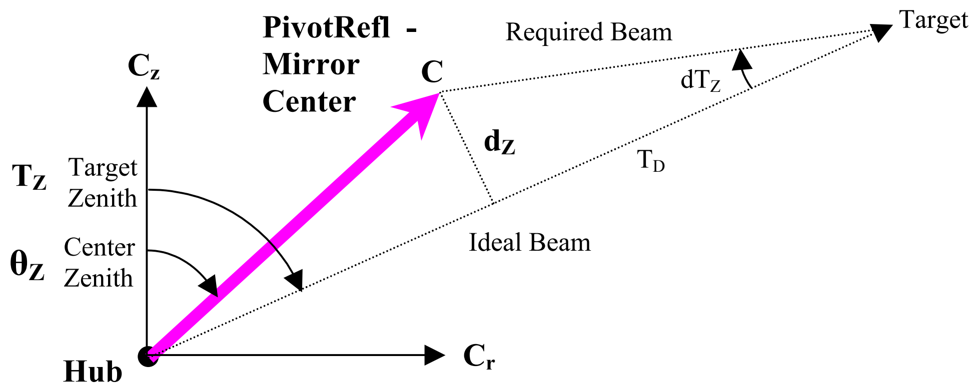


Figure 5: Free-Body Diagram of Elevation Offset

The relative offset, $\mathbf{d_Z}$, is determined by:

$\mathbf{\theta_Z = Atan\ (C_r\ /\ C_z)}$
$\mathbf{d_Z = C * Sin\ (T_Z - \theta_Z)}$

Then the angular offset, $\mathbf{dT_Z}$, determined by:

$\mathbf{T_D = Sqrt\ (Target\_X^2 + Target\_Y^2 + Target\_Z^2)}$
$\mathbf{dT_Z = Atan\ (d_Z\ /\ (T_D - C * Sin\ (T_Z - \theta_Z)))}$

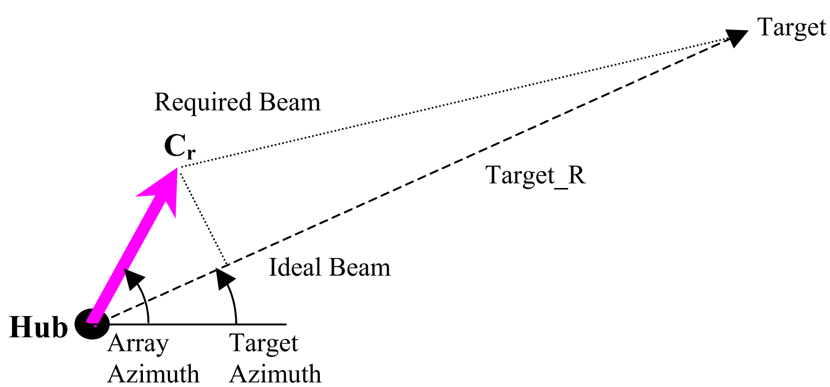


Figure 6: Free-Body Diagram of Azimuth Offset

Appendix A:

Equation of Apparent Sun Motion

Compute Greenwich Mean Time:

GMT = Zone + Hour + (Min / 60.0) + (Sec/3600.0)

Compute the Julian Date:

JD = (367 * Year - (7 * (Year + ((Mon + 9) / 12)) / 4) +
(275 * Mon / 9) + Day) + 1721013.5 + **GMT** / 24.0

Compute 'Little T': The number of days from 2000 AD. (was negative until 2000)

t = **JD** - 2451545.0

Compute 'Big T': Centuries since 1900 AD.

(FRC takes the fractional portion of the result and discards the whole numbers)

T = 1.0 + **t** / 36525.0

$\mathbf{\Omega_M}$ = 2π * FRC (5.347343 - 0.00014709391 * **t**)

$\mathbf{L_S}$ = 2π * FRC (11.779072 + 0.00273790931 * **t**)

$\mathbf{L_M}$ = 2π * FRC (150.606434 + 0.03660110129 * **t**)

$\mathbf{G_S}$ = 2π * FRC (10.993126 + 0.00273777850 * **t**)

$\mathbf{G_2}$ = 2π * FRC (18.140023 + 0.00445036173 * **t**)

$\mathbf{G_4}$ = 2π * FRC (6.053856 + 0.00145561327 * **t**)

$\mathbf{G_5}$ = 2π * FRC (1.056531 + 0.00023080893 * **t**)

E = **(**84428.0 - 47.0 * **T** + 9.0 * Cos ($\mathbf{\Omega_M}$)**)** / **(**3600 * 180) / **π)**

dλ = -17.0 * Sin (**ΩM**) / ((3600 * 180) / **π**)

ST = 0.2769194 + (1.07523148e-6 * **T** + 100.002136) * **T**

GST = FRC (**ST** + **GMT** / 24.0)

Legend for following equations:
rLat, **rLong** = Latitude, Longitude in radians

P_L = [6910.0 * Sin (G_S) + 72.0 * Sin (2.0 * G_S) - 17.0 * **T** * Sin (G_S)
- 7.0 * Cos ($G_S - G_5$) + 6.0 * Sin ($L_M - L_S$)
+ 5.0 * Sin (4.0 * G_S - 8.0 * G_4 + 3.0 * G_5)
- 5.0 * Cos (2.0 * G_S - 2.0 * G_2) - 4.0 * Sin ($G_S - G_2$)
+ 4.0 * Cos (4.0 * G_S - 8.0 * G_4 + 3.0 * G_5)
+ 3.0 * Sin (2.0 * G_S - 2.0 * G_2) - 3.0 * Sin (G_5)
- 3.0 * Sin (2.0 * G_S - 2.0 * G_5)] / [3600 * 180 / π]

$\lambda = P_L + L_S$

$L_{SUN} = (\lambda + d\lambda)$

Δ = Asin (Sin (L_{SUN}) * Sin (**E**))

RA = Acos (Cos (L_{SUN}) / Cos (Δ))

If (Δ < 0.0) **RA** = 2π - **RA**

HA = 2π * FRC (2.0 + (**rLong** / 2π + **RA** / 2π - **GST**))

If (**HA** > π) **HA** = **HA** - 2π

Cos (**rLat**) * Cos (Δ) * Cos (**HA**))

α_z = Acos (Sin (**rLat**) * Sin (Δ) + Cos (**rLat**) * Cos (Δ) * Cos (**HA**))

```
If (αz < 2.0e-8) {
    αz = 0.0
    If (HA <= 0.0) αa = π / 2
      Else αa = -π / 2
 }
  Else {
        αa = Cos (Δ) * Sin (HA) / Sin (αz)
        If (αa >= 1.0) αa = π / 2
            Else If (αa <= -1.0) αa = -π / 2
                     Else αa = Asin (αa)

    If ((Sin (Δ) / Sin (rLat) - Cos (αz)) > 0.0) {
        If (αa >= 0.0) αa = π - αa
            Else αa = -αa - π
      }
```

